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Vibration suppression in belt-driven servo systems containing uncertain nonlinear dynamics ☆

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ABSTRACT

This paper proposes a vibration suppression strategy for belt-driven servo systems subject to uncertain nonlinear dynamics and external disturbances. The function approximation technique is applied to estimate the uncertainties that are further covered by sliding-based design. The closed loop stability is justified with Lyapunov-like method to ensure ultimately uniformly bounded performance of the output error. Simulation cases show that the proposed strategy can stabilize the closed loop system with effective suppression of vibration regardless of various uncertain nonlinear dynamics and external disturbances.

1. Introduction

Weighted synchronization of neighboring axes can be done by couplings of gears. If the distance between axes is large, two popular synchronization techniques are available: the e-gear design and the belt system. With the advancement of the mechatronics technology, traditional mechanical computations using such as gear sets, linkages and cam mechanisms are able to be replaced by electronic computations. The e-gear concept is based on the broadcasting of the master axis motion information via communication links so that distant axes can be synchronized by driving motors with those master motion trajectories as the reference signal [1–4]. Since there is no mechanical coupling between the master and slaves, long distance synchronization among multiple axes can be realized. In this configuration, each slave axis can be with different loading conditions and external disturbances. Individual robust or adaptive strategies can be designed to ensure good tracking performance of each axis and hence the synchronization performance of the entire system is obtained.

In general, the e-gear system lacks of one very important property in the mechanical synchronization designs, i.e., the loading conditions of the slaves cannot be reflected back to the master directly. Although some more sophisticated designs with feedback paths from slaves can be constructed to this purpose, regulation for consistent global synchronization performance is still a challenge.

The flat belt mechanism is widely used in industrial applications for long distance power transmission due to its low cost and low noise. Since its operation is mainly via the friction force between the belt and pulleys, the slipping effect largely reduces the synchronization performance [5]. On the other hand, the timing belt effectively eliminates the slipping which results in more applications requiring high speed and high efficiency [6]. For precision servo applications, however,

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compliance of the belt may induce mechanical vibration or high-order resonance that will reduce system stability margin in the servo loop [7–10]. The flexibility of the belt is also a known source of nonlinearity in the closed loop dynamics. In addition, the dynamics of the belt may change due to belt aging and environmental factors such as temperature and humidity [11]. Differences in the loading condition will also give variations in its dynamic behavior. Therefore, belt-driven servo systems may only give limited repeatability and accuracy.

To improve the control performance, Jokinen et al. [6] presented a detailed report on the physical drawbacks in high speed belt driven systems. Better modeling techniques were proposed in Gong et al. [12] and Huang et al. [5] to facilitate the controller design. Many active control strategies have been proposed to suppress vibrations and improve servo performance in the belt-driven servo systems under various system uncertainties. Li and Rehani [13] suggested a PID controller with some tuning rule for positioning problems in belt-driven systems. Many intelligent and learning based controller designs were also suggested [9,11,14–16]. Various robust designs were reported in Hace et al. [7,8,17,18], and Sabanovic et al. [19]. Wu et al. [10] also proposed an input shaping technique to reduce residual vibration in the belt-driven systems.

Because it is very difficult to have a precise belt dynamics, model-based control strategies can only give limited performance when accurate positioning is required. To eliminate the inherent vibration due to the belt dynamics, some more advanced strategies are needed. From the review presented above, few reports on the adaptive designs for the control of belt-driven systems are found which is mainly because there are too many time dependent uncertainties in the system dynamics. In this paper, we would like to employ the function approximation technique (FAT) based strategy [20–29] to propose an adaptive controller to effectively suppress vibrations in an uncertain belt-driven servo system so that good performance can be achieved. The highly nonlinear belt dynamics is firstly represented as a finite combination of known basis functions. The coefficient vector is then updated by an adaptation algorithm. To prevent possible parameter drifting, the σ-modifications to the update laws are enforced. The steady-state behavior of the system output error can be proved to be uniformly ultimately bounded by using the Lyapunov-like stability theory. Simulation cases are presented to justify the proposed strategy.

This paper is organized as follows. Section 2 introduces the equation of motion of a belt-driven servo system. Section 3 derives the function approximation based adaptive controller for vibration suppression. Section 4 presents the simulation results. The last section concludes the paper.

2. Dynamic model of belt-driven servo systems

A typical belt-driven servo system is shown in Fig. 1. A servo motor is installed to drive the driving pulley on the left so that the table M_c can be regulated to the desired position precisely. The symbols K_1, K_2 and K_3 represent the nonlinear spring effect of the belt flexibility. The actual spring forces for these three nonlinear springs are modeled as $k_{i1}(x)x+k_{i2}(x)x^3$, $i=1,2,3$, where $k_{ij}(x)$ are state-dependent stiffness coefficients. Due to these flexible components, unwanted vibration could be excited if the system is not properly controlled.

The mathematical model of this system can easily be found as

$$\begin{aligned} &M_c\ddot{x}-k_{11}(Rq_1-x)-k_{12}(Rq_1-x)^3+k_{21}(x-Rq_2)\\ &\quad+k_{22}(x-Rq_2)^3+f_f+f_d=0\\ &J_2\ddot{q}_2-k_{21}R(x-Rq_2)-k_{22}R(x-Rq_2)^3+k_{31}R^2(q_2-q_1)\\ &\quad+k_{32}R^2(q_2-q_1)^3+\tau_{f2}=0\\ &[J_1+G^2(J_G+J_m)]\ddot{q}_1+k_{11}R(Rq_1-x)+k_{12}R(Rq_1-x)^3\\ &\quad+k_{31}R^2(q_2-q_1)+k_{32}R^2(q_2-q_1)^3+\tau_{f1}=G\tau \end{aligned} \tag{1}$$

where $x\in\Re$ is the displacement of the table whose mass is denoted as M_c. The real numbers q_1 and q_2 are, respectively, the angular displacements of the master and idle pulley. Assume that both pulleys are with the same radius, but with different inertia where J_1 is for the master while J_2 is for the idle. The motor output torque is denoted as τ, and the positive numbers G represents the gear ratio in the velocity reduction mechanism. J_m and J_G are, respectively, the inertia of the motor and the gear set. τ_{f1} and τ_{f2} are torques induced by friction forces in the two pulleys, respectively, while f_f is the friction between the table and its support. The symbol f_d represents the external disturbance on the table.

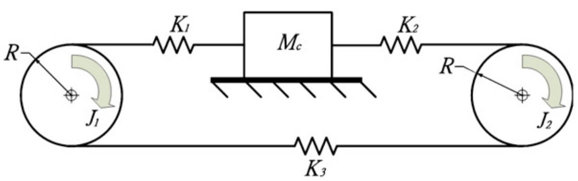

Fig. 1. A typical belt-driven servo system.

Let us rewrite system (1) in its state space representation by defining

$$\mathbf{x}=\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{bmatrix}^T = \begin{bmatrix} x & \dot{x} & q_2 & \dot{q}_2 & q_1 & \dot{q}_1 \end{bmatrix}^T \in \Re^6$$

and the system equation becomes

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= x_3+d_2(\mathbf{x})\\ \dot{x}_3 &= x_4\\ \dot{x}_4 &= x_5+d_4(\mathbf{x})\\ \dot{x}_5 &= x_6\\ \dot{x}_6 &= d_6(\mathbf{x})+gu\end{aligned} \tag{2}$$

where

$$\begin{aligned} d_2(\mathbf{x}) &= -x_3+\frac{k_{11}}{M_c}(Rx_5-x_1)+\frac{k_{12}}{M_c}(Rx_5-x_1)^3-\frac{k_{21}}{M_c}(x_1-Rx_3)\\ &\quad -\frac{k_{22}}{M_c}(x_1-Rx_3)^3-\frac{1}{M_c}f_f-\frac{1}{M_c}f_d\\ d_4(\mathbf{x}) &= -x_5+\frac{k_{21}}{J_2}R(x_1-Rx_3)+\frac{k_{22}}{J_2}R(x_1-Rx_3)^3-\frac{k_{31}}{J_2}R^2(x_3-x_5)\\ &\quad -\frac{k_{32}}{J_2}R^2(x_3-x_5)^3-\frac{1}{J_2}\tau_{f2}\\ d_6(\mathbf{x}) &= \frac{1}{J_1+G^2(J_G+J_m)}\Big[-k_{11}R(Rx_5-x_1)-k_{12}R(Rx_5-x_1)^3\\ &\quad -k_{31}R^2(x_3-x_5)-k_{32}R^2(x_3-x_5)^3-\tau_{f1}\Big]\\ g &= \frac{G}{J_1+G^2(J_G+J_m)}\\ u &= \tau \end{aligned}$$

Since the spring stiffness and the friction forces are difficult to obtain, we would like to assume that their values are unavailable. Therefore, d_2, d_4 and d_6 become uncertainties. It should be noted that g is a constant and is also assumed to be unknown. Let us represent it in the multiplicative form $g=g_m\Delta g$ where g_m is the known nominal value and Δg is the uncertainty satisfying

$$\delta_{\min} \equiv \frac{g_{\min}}{g_m} \le \Delta g \le \frac{g_{\max}}{g_m} \equiv \delta_{\max} \tag{3}$$

where $g_{\min}$ and $g_{\max}$ are, respectively, the lower bound and upper bound for g. Since uncertainties d_2, d_4 and d_6 are functions of states, their values are time-varying without knowing their variation bounds; therefore, both conventional adaptive designs and robust strategies are not feasible. What is worse is that d_2 and d_4 enter the system in a mismatched fashion; therefore, most control schemes fail. In next section, we are going to design an FAT based adaptive controller to stabilize the whole system without knowing its precise model. Besides, the vibration in the system output should be effectively suppressed.

3. System stabilization and vibration suppression

In this section, we would like to design a controller u for system (2) such that the error between system output x_1 and its desired value x_{1d} is small with rigorous mathematical justification. The derivation is based on the sliding control theory incorporating with the adaptive design so that the mismatched uncertainties can be tolerated. Since the order of system (2) is 6, let us define the error signals below to quantify the convergence performance

$$e_i = x_i - x_{id}, \quad i=1,\ldots,6$$

where x_{id} is the desired trajectory of x_i. A step-by-step design procedure based on Huang and Chen [24] is employed. Firstly, let us consider the error dynamics of e_1 by taking the time derivative of $e_1=x_1-x_{1d}$ and plugging the relationship $\dot{x}_1=x_2$ and $x_2=e_2+x_{2d}$ to have

$$\dot{e}_1 = e_2 + x_{2d} - \dot{x}_{1d} \tag{4}$$

It is seen that the dynamics of e_1 is driven by the right-hand side of Eq. (4) where $\dot{x}_{1d}$ is known and x_{2d} is to be designed. To stabilize this dynamics, we may regard x_{2d} as a virtual control, and we prefer a realizable design as

$$x_{2d} = \dot{x}_{1d} - c_1 e_1 \tag{5}$$

where c_1 is a positive number. With this selection, Eq. (4) can be rewritten as

$$\dot{e}_1 = e_2 - c_1 e_1 \tag{6}$$

Obviously, if some proper control strategies are available such that e_2 is small, then e_1 will be small as desired. Therefore, we need to investigate the dynamics of e_2. Along the same line, the next step is to take the time derivative of e_2 as

$$\dot{e}_2 = e_3 + x_{3d} + d_2 - \dot{x}_{2d} \tag{7}$$

where $\dot{x}_{2d}$ is a known value which can be found from Eqs. (5) and (4). Similarly, we would like to make e_2 small by stabilizing this dynamics. Now, let us regard x_{3d} as a virtual control to be in the form

$$x_{3d} = \dot{x}_{2d} - \hat{d}_2 - c_2 e_2 \tag{8}$$

where c_2 is a positive number, and $\hat{d}_2$ is an estimate of the mismatched uncertainty d_2. Insert Eqs. (8) into (7), and we may have

$$\dot{e}_2 = e_3 - c_2 e_2 + (d_2 - \hat{d}_2) \tag{9}$$

This error dynamics implies that if e_3 is small and some update law exists such that the error between d_2 and $\hat{d}_2$ is small then e_2 will also be small. Together with Eq. (6), we may further have small magnitude in e_1. However, to ensure e_3 small, we need to stabilize its dynamics. To this end, let us find its error dynamics by taking the time derivative as

$$\dot{e}_3 = e_4 + x_{4d} - \dot{x}_{3d} \tag{10}$$

where $\dot{x}_{3d}$ can be found from Eqs. (8) and (9) as

$$\begin{aligned} \dot{x}_{3d} &= \ddot{x}_{2d} - \dot{\hat{d}}_2 - c_2(e_3 - c_2 e_2 + d_2 - \hat{d}_2) \\ &\equiv \dot{x}_{3dk} + \dot{x}_{3du} \end{aligned}$$

where $\dot{x}_{3dk} = \ddot{x}_{2d} - \dot{\hat{d}}_2 - c_2(e_3 - c_2 e_2 - \hat{d}_2)$ is the known part in $\dot{x}_{3d}$, while $\dot{x}_{3du} = -c_2 d_2$ is the unknown part in $\dot{x}_{3d}$. Let us define $\overline{d}_3 \equiv -\dot{x}_{3du}$, then we may select x_{4d} as

$$x_{4d} = \dot{x}_{3dk} - \hat{\overline{d}}_3 - c_3 e_3 \tag{11}$$

where $\hat{\overline{d}}_3$ is an estimate of $\overline{d}_3$, and c_3 is a positive number. This way the error dynamics of e_3 becomes

$$\dot{e}_3 = e_4 - c_3 e_3 + (\overline{d}_3 - \hat{\overline{d}}_3) \tag{12}$$

Likewise, if e_4 is small and we may design some update law to have $\overline{d}_3 - \hat{\overline{d}}_3$ small, then small magnitude in e_3 can be ensured. Next, we investigate the dynamics of e_4 as

$$\begin{aligned} \dot{e}_4 &= e_5 + x_{5d} + d_4 - \dot{x}_{4d} \\ &\equiv e_5 + x_{5d} + d_4 - \dot{x}_{4dk} - \dot{x}_{4du} \end{aligned} \tag{13}$$

where $\dot{x}_{4dk}$ and $\dot{x}_{4du}$ are, respectively, the known and unknown part of $\dot{x}_{4d}$ that can be easily found from Eqs. (11) and (12). Define $\overline{d}_4 \equiv d_4 - \dot{x}_{4du}$, then we may select x_{5d} as

$$x_{5d} = \dot{x}_{4dk} - \hat{\overline{d}}_4 - c_4 e_4 \tag{14}$$

where $\hat{\overline{d}}_4$ is an estimate of $\overline{d}_4$, and c_4 is a positive number. Therefore, the error dynamics of e_4 becomes

$$\dot{e}_4 = e_5 - c_4 e_4 + (\overline{d}_4 - \hat{\overline{d}}_4) \tag{15}$$

Along the same line, we may find the error dynamics of e_5 to be

$$\begin{aligned} \dot{e}_5 &= e_6 + x_{6d} - \dot{x}_{5d} \\ &\equiv e_6 + x_{6d} - \dot{x}_{5dk} - \dot{x}_{5du} \\ &\equiv e_6 + x_{6d} - \dot{x}_{5dk} + \overline{d}_5 \end{aligned} \tag{16}$$

The desired trajectory x_{6d} can thus be determined as

$$x_{6d} = \dot{x}_{5dk} - \hat{\overline{d}}_5 - c_5 e_5 \tag{17}$$

Then we have

$$\dot{e}_5 = e_6 - c_5 e_5 + (\overline{d}_5 - \hat{\overline{d}}_5) \tag{18}$$

Finally, the error dynamics for e_6 is found to be

$$\begin{aligned} \dot{e}_6 &= d_6 + gu - \dot{x}_{6dk} - \dot{x}_{6du} \\ &\equiv \overline{d}_6 + gu - \dot{x}_{6dk} \end{aligned} \tag{19}$$

To stabilize this error dynamics, we may select the control law as

$$u = \frac{1}{g_m}(-\hat{\overline{d}}_6 + \dot{x}_{6dk} - c_6 e_6 - u_r) \tag{20}$$

where u_r is a robust term to be determined. With this design of the control law, Eq. (19) can be further derived as

$$\dot{e}_6 = -c_6 e_6 + (\overline{d}_6 - \hat{\overline{d}}_6) + (1-\Delta g)(\hat{\overline{d}}_6 - \dot{x}_{6dk} + c_6 e_6) - \Delta g u_r \tag{21}$$

At this stage, we have already obtained the dynamics for all error signals, and they can be collected from Eqs. (6), (9), (12), (15), (18) and (21) to form

$$\begin{aligned}\dot{e}_1 &= e_2 - c_1 e_1\\ \dot{e}_2 &= e_3 - c_2 e_2 + (\overline{d}_2 - \hat{\overline{d}}_2)\\ \dot{e}_3 &= e_4 - c_3 e_3 + (\overline{d}_3 - \hat{\overline{d}}_3)\\ \dot{e}_4 &= e_5 - c_4 e_4 + (\overline{d}_4 - \hat{\overline{d}}_4)\\ \dot{e}_5 &= e_6 - c_5 e_5 + (\overline{d}_5 - \hat{\overline{d}}_5)\\ \dot{e}_6 &= -c_6 e_6 + (\overline{d}_6 - \hat{\overline{d}}_6) + (1-\Delta g)(\hat{\overline{d}}_6 - \dot{x}_{6dk} + c_6 e_6) - \Delta g u_r\end{aligned} \tag{22}$$

where we have defined $\overline{d}_2 = d_2$ and $\hat{\overline{d}}_2 = \hat{d}_2$ for simplification in derivation. From here, it is clear that if update laws for $\hat{\overline{d}}_i$, i=2,…,6, are designed properly and the robust term u_r is effective to cover the terms in the last subsystem in Eq. (22) involving the multiplicative uncertainty, then we may conclude that the output error will be small in its magnitude. Hence, we would like to design the update laws and robust term by using the FAT and the Lyapunov-like method.

Since we assume that $\overline{d}_i$, i=2,…,6, are time-varying uncertainties, we may not apply conventional adaptive strategies to derive update laws. Now, we would like to approximate their values by using orthonormal basis with the form

$$\left.\begin{aligned}\overline{d}_i &= \mathbf{w}_i^T \mathbf{z}_i + \varepsilon_i\\ \hat{\overline{d}}_i &= \mathbf{w}_i^T \mathbf{z}_i\end{aligned}\right., \quad i = 2,\ldots,6 \tag{23}$$

where $\mathbf{w}_i$ is a vector of unknown coefficients, $\mathbf{z}_i$ is a vector of known basis functions, and ε_i represents the approximation error. Define $\tilde{\mathbf{w}}_i = \mathbf{w}_i - \hat{\mathbf{w}}_i$, and Eq. (22) becomes

$$\begin{aligned}\dot{e}_1 &= e_2 - c_1 e_1\\ \dot{e}_2 &= e_3 - c_2 e_2 + \tilde{\mathbf{w}}_2^T \mathbf{z}_2 + \varepsilon_2\\ \dot{e}_3 &= e_4 - c_3 e_3 + \tilde{\mathbf{w}}_3^T \mathbf{z}_3 + \varepsilon_3\\ \dot{e}_4 &= e_5 - c_4 e_4 + \tilde{\mathbf{w}}_4^T \mathbf{z}_4 + \varepsilon_4\\ \dot{e}_5 &= e_6 - c_5 e_5 + \tilde{\mathbf{w}}_5^T \mathbf{z}_5 + \varepsilon_5\\ \dot{e}_6 &= -c_6 e_6 + \tilde{\mathbf{w}}_6^T \mathbf{z}_6 + \varepsilon_6 + (1-\Delta g)(\hat{\overline{d}}_6 - \dot{x}_{6dk} + c_6 e_6) - \Delta g u_r\end{aligned} \tag{24}$$

Define a Lyapunov-like function candidate for the last equation in Eq. (24) as

$$V_6 = \tfrac{1}{2} e_6^2 + \tfrac{1}{2} \tilde{\mathbf{w}}_6^T \mathbf{\Gamma}_6 \tilde{\mathbf{w}}_6 \tag{25}$$

where $\mathbf{\Gamma}_6$ is a positive definite weighting matrix with proper dimension. The time derivative of V_6 along the trajectory of Eq. (24) is found as

$$\dot{V}_6 \le -c_6 e_6^2 + e_6 \varepsilon_6 + (1+\delta_{\max}) \left| \hat{\overline{d}}_6 - \dot{x}_{6dk} + c_6 e_6 \right| |e_6| - \delta_{\min} e_6 u_r + \tilde{\mathbf{w}}_6^T (e_6 \mathbf{z}_6 - \mathbf{\Gamma}_6 \dot{\hat{\mathbf{w}}}_6) \tag{26}$$

Hence, we may design the robust term and the update law as

$$u_r = \frac{1+\delta_{\max}}{\delta_{\min}} \left| \hat{\overline{d}}_6 - \dot{x}_{6dk} + c_6 e_6 \right| \operatorname{sgn}(e_6) \tag{27a}$$

$$\dot{\hat{\mathbf{w}}}_6 = \mathbf{\Gamma}_6^{-1} (e_6 \mathbf{z}_6 - \sigma_6 \hat{\mathbf{w}}_6) \tag{27b}$$

where σ_6 is a positive constant for the σ-modification of the update law to prevent possible parameter drifting. Therefore, Eq. (26) becomes

$$\dot{V}_6 \le -c_6 e_6^2 + |e_6||\varepsilon_6| + \sigma_6 (\tilde{\mathbf{w}}_6^T \mathbf{w}_6 - \|\tilde{\mathbf{w}}_6\|^2) \tag{28}$$

By using the inequalities

$$\begin{aligned}-c_6 e_6^2 + |e_6||\varepsilon_6| &\le -\frac{1}{2}\left(c_6 e_6^2 - \frac{\varepsilon_6^2}{c_6}\right)\\ \tilde{\mathbf{w}}_6^T \mathbf{w}_6 - \|\tilde{\mathbf{w}}_6\|^2 &\le -\frac{1}{2}(\|\tilde{\mathbf{w}}_6\|^2 - \|\mathbf{w}_6\|^2)\end{aligned} \tag{29}$$

Eq. (28) can be derived as

$$\dot{V}_6 \le -\frac{1}{2}\left(c_6 e_6^2 - \frac{\varepsilon_6^2}{c_6}\right) - \frac{1}{2}\sigma_6 (\|\tilde{\mathbf{w}}_6\|^2 - \|\mathbf{w}_6\|^2)$$

$$\le -\alpha_6 V_6+\alpha_6\left[\frac{e_6^2}{2}+\frac{1}{2}\lambda_{\max}(\boldsymbol{\Gamma}_6)\|\tilde{\mathbf{w}}_6\|^2\right]-\left[\frac{c_6 e_6^2}{2}+\frac{1}{2}\sigma_6\|\tilde{\mathbf{w}}_6\|^2\right]+\frac{1}{2}\sigma_6\|\mathbf{w}_6\|^2+\frac{\varepsilon_6^2}{2c_6}$$

where α_6 is selected as $\alpha_6 \le \min\{c_6,(\sigma_6/\lambda_{\max}(\boldsymbol{\Gamma}_6))\}$ to give

$$\dot{V}_6 \le -\alpha_6 V_6+\frac{1}{2}\sigma_6\|\mathbf{w}_6\|^2+\frac{1}{2c_6}\varepsilon_6^2 \tag{30}$$

Therefore, $\dot{V}_6<0$ whenever

$$(e_6,\tilde{\mathbf{w}}_6)\in\left\{(e,\tilde{\mathbf{w}}_6)\middle|V_6>\phi_6\equiv\frac{\sigma_6}{2\alpha_6}\|\mathbf{w}_6\|^2+\frac{1}{2\alpha_6 c_6}\sup\varepsilon_6^2(\tau)\right\} \tag{31}$$

where ϕ_6 can be regarded as the thickness of the boundary layer for the 6th error signal. The result obtained here implies that $(e_6,\tilde{\mathbf{w}}_6)$ is uniformly ultimately bounded. In addition, we can conclude that given any $\mu_6>0$, there exist $T_6\ge t_0\ge 0$ such that

$$V_6(t)\le\phi_6+\mu_6 \quad \text{for } \tau\ge T_6 \tag{32}$$

To facilitate the following derivation, we define $\tilde{\mathbf{w}}_1=0$ and $\varepsilon_1=0$ (they do not appear in Eq. (24)). Then, we may define the Lyapunov-like functions

$$V_i=\tfrac{1}{2}e_i^2+\tfrac{1}{2}\tilde{\mathbf{w}}_i^T\boldsymbol{\Gamma}_i\tilde{\mathbf{w}}_i,\quad i=5,4,3,2,1 \tag{33}$$

With the time derivative along the system trajectory as

$$\dot{V}_i=-c_i e_i^2+e_i(e_{i+1}+\varepsilon_i)+\tilde{\mathbf{w}}_i^T(e_i\mathbf{z}_i-\boldsymbol{\Gamma}_i\dot{\hat{\mathbf{w}}}_i) \tag{34}$$

we may design the update laws to be

$$\dot{\hat{\mathbf{w}}}_i=\boldsymbol{\Gamma}_i^{-1}(e_i\mathbf{z}_i-\sigma_i\hat{\mathbf{w}}_i) \tag{35}$$

where σ_i is a positive number. After some rearrangement, Eq. (34) becomes

$$\dot{V}_i\le-\alpha_i V_i+\frac{\sigma_i}{2}\|\mathbf{w}_i\|^2+\frac{1}{2c_i}(\varepsilon_i+e_{i+1})^2 \tag{36}$$

where $\alpha_i\le\min\{c_i,(\sigma_i/\lambda_{\max}(\boldsymbol{\Gamma}_i))\}$ For $t_0\le t<T_{i+1}$, we may have $\dot{V}_i<0$ whenever

$$(e_i,\tilde{\mathbf{w}}_i)\in E_i\equiv\{(e_i,\tilde{\mathbf{w}}_i)|V_i>\phi_i\} \tag{37}$$

where

$$\phi_i\equiv\frac{\sigma_i}{2\alpha_i}\|\mathbf{w}_i\|^2+\frac{1}{2\alpha_i c_i}\left[\sup_{\tau\ge t_0}|\varepsilon_i(\tau)|+\sqrt{2\max\left\{V_{i+1}(t_0),\quad\phi_{i+1}+\mu_{i+1}\right\}}\right]^2$$

This implies that V_i is bounded for all $t\in[t_0,T_{i+1}]$, i.e., before convergence of V_{i+1}, V_i is bounded. Define

$$\phi'_i\equiv\frac{\sigma_i}{2\alpha_i}\|\mathbf{w}_i\|^2+\frac{1}{2\alpha_i c_i}\left[\sup_{\tau\ge t_0}|\varepsilon_i(\tau)|+\sqrt{2(\phi_{i+1}+\mu_{i+1})}\right]^2$$

then after the convergence of V_{i+1} (i.e., $t\ge T_{i+1}$) into $\phi'_{i+1}+\mu_{i+1}$, V_i is bounded where $\mu_i>0$ is some constant such that there exist $T_i\ge T_{i+1}$ to have $V_i\le\phi'_i+\mu_i$ for all $t\ge T_i$. We have proved that $(e_i,\tilde{\mathbf{w}}_i)$ is uniformly ultimately bounded, and also established the order of convergence starting from e_n and ending with e_1. During convergence of e_i, boundedness of e_j, $j=i-1,\ldots,1$ are ensured. Specifically, when $t\ge T_1$, we may have

$$|e_1(t)|=|x_1-x_{1d}|=\sqrt{2V_1}\le\sqrt{2(\phi'_1+\mu_1)} \tag{38}$$

This ensures that the output error of the belt servo system with the proposed controller will be bounded by some constant adjustable by controller parameters. It is shown that the performance is achieved regardless of the mismatched time-varying uncertainties.

Theorem. *Consider the belt-driven servo system described in* (2), *where* d_2, d_4 *and* d_6 *are state dependent uncertainties whose variation bounds are not available, and g is a bounded uncertainty satisfying* (3). *By designing the robust term according to* (27a), *the control strategy* (20) *together with the update laws* (27b) and (35) *ensure that the system output be uniformly ultimately bounded. Specifically, the output signal is upper bounded according to* (38), *which is adjustable by controller parameters.*

4. Simulation results

To verify the vibration suppression ability of the proposed design, two simulation cases are presented for the belt-driven servo system in Fig. 1. The first case is the regulation of the table starting from rest at the initial position $x_1=0(m)$ to

x_d=1(m) with time-varying disturbances. The second case presents the same regulation problem but the external disturbance is not only time-varying but also state-dependent. For both cases, the PID control performance for vibration suppression will be given for comparison. The system parameters used in the simulation are selected as M_c=30 kg, k_{11}=490,000 N/m, k_{12}=150,000 N/m^3, k_{21}=260,000 N/m, k_{22}=120,000 N/m^3, k_{31}=170,000 N/m, k_{32}=130,000 N/m^3, G=1, R=0.02 m, J_1=J_2=0.000035 kg m^2, J_G=0.00012 kg m^2 and J_m=0.0012 kg m^2. A lumped friction model [30] is used to represent the friction force between the table and its support as

$$f_f = F_c \operatorname{sgn}(\dot{x}) + (F_s - F_c)\operatorname{sgn}(\dot{x}) e^{-(\dot{x}/V_S)^2} + C_v \dot{x}$$

where F_s is the static friction effect, F_c is the Coulomb friction force, C_v is the viscous friction parameter and V_s is the characteristic velocity of the Stribeck friction. In practical applications, except for the static friction force F_s which can be easily obtained by simple experiments, all other objects in this model are generally not readily available. On the other hand, the friction effect depends on the table position, lubrication condition, temperature, operation situation, etc.; therefore, it is reasonable to model its effect as a function of the system states and time. In this simulation study, we use F_c=35 N, C_v=15 N s/m, F_s=0 N and V_s=0.019 m/s. Friction forces between the belt and pulleys are modeled as $\tau_{f1} = 0.05\dot{q}_2$ (N) and $\tau_{f2} = 0.05\dot{q}_1$ (N). The controller parameters used in both cases are selected as c_1=8.5, c_2=0.3, c_3=5.0, c_4=0.01, c_5=1.5, c_6=0.5, σ_1=5, σ_2=0.0000001, σ_3=6.2, σ_4=3, σ_5=7, σ_6=0.6, $\mathbf{\Gamma}_1 = 0.136\mathbf{I}$, $\mathbf{\Gamma}_2 = \mathbf{\Gamma}_3 = \mathbf{\Gamma}_4 = \mathbf{\Gamma}_5 = 0.003\mathbf{I}$ and $\mathbf{\Gamma}_6 = 0.001\mathbf{I}$. The 11-term Fourier series is used as the basis for function approximation.

Case 1: Regulation with time-varying disturbance.

In this case, we consider the regulation of the table position subject to time-varying disturbance $f_d = 10 + (-5\sin 2t)^3$ (N). The time history of the table displacement by using the PID controller is shown in Fig. 2. It is seen that significant vibration is observed in the transient phase, while some low frequency oscillations still present in the steady state. In addition, to stabilize the system, the PID controller has to send out large energy. On the other hand, the FAT-based controller drives the table smoothly to the target position without any oscillation in Fig. 3 regardless of the system uncertainties and external disturbances. Besides, the transient is much faster than that of the PID control. The steady-state error here is 0.1431 mm. The control effort is typical to sliding based strategies. Since the mechanical part of the system is a low-pass filter, it can effectively eliminate the high frequency component in the control signal resulting in a smooth trajectory in the table displacement. The fast control activity can be realized with switching power amplification techniques easily.

Case 2: Regulation with time-varying and state-dependent disturbance.

The disturbance considered in this case is $f_d = \sin x + x \sin 5t$ which is applied to the table directly. Fig. 4 shows that the PID controller cannot give stable regulation in this case. The proposed controller can still give consistent performance both in the transient and steady state. Therefore, we may say that the proposed strategy can give good performance to the belt-driven servo systems without excitation of vibration (Fig. 5).

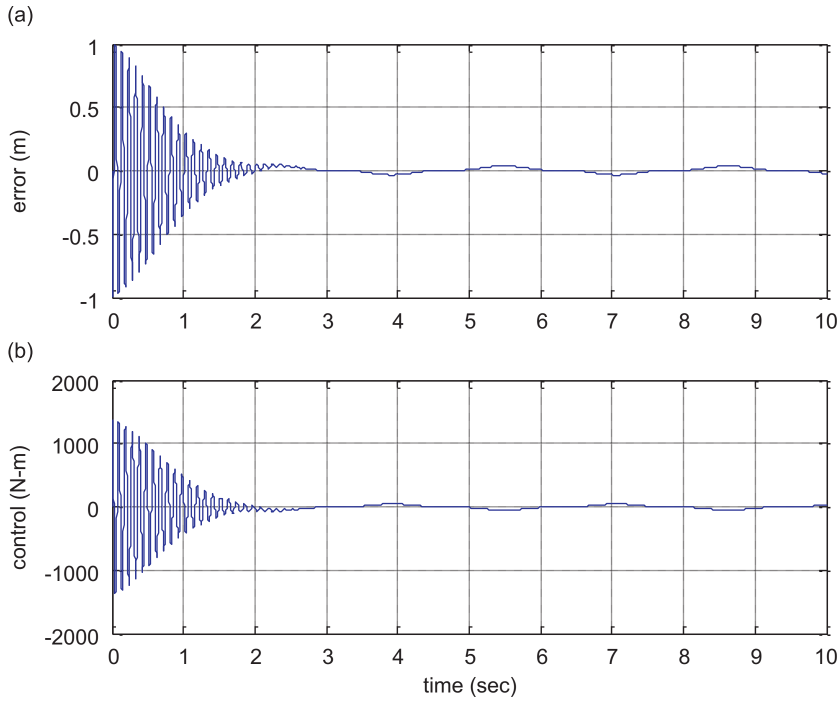

Fig. 2. PID control performance in case 1: (a) output error and (b) control effort.

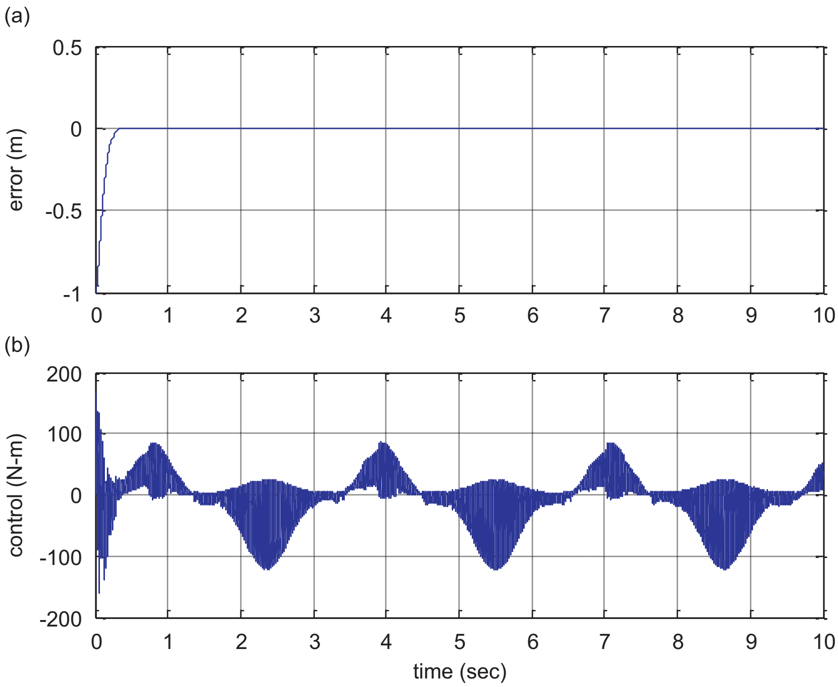

Fig. 3. Proposed control performance in case 1: (a) output error and (b) control effort.

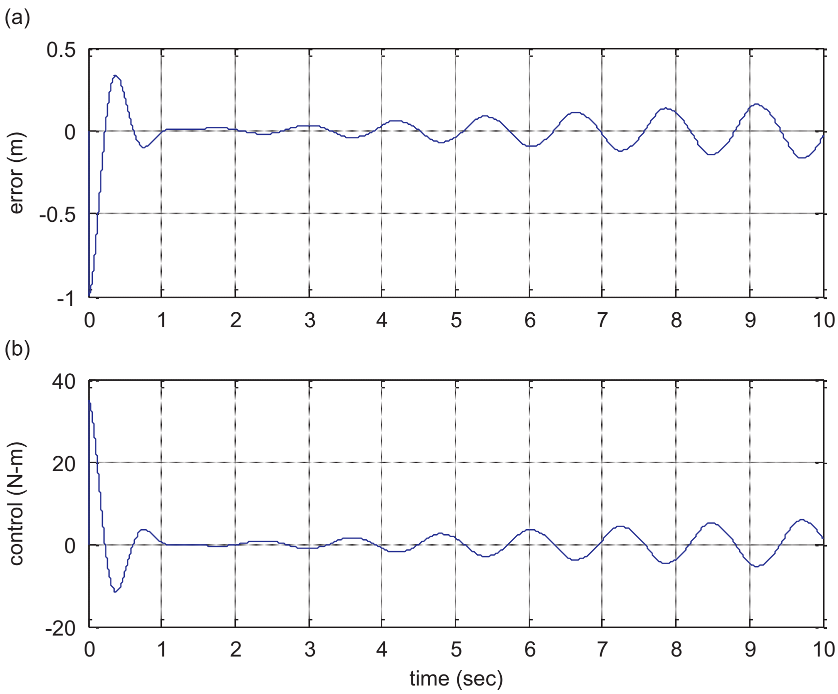

Fig. 4. PID control performance in case 2: (a) output error and (b) control effort.

5. Conclusions

An active control strategy is proposed to a nonlinear belt-driven servo system with time-varying uncertainties subject to external disturbances to achieve fast positioning without excitation of vibration. The strategy is based on the

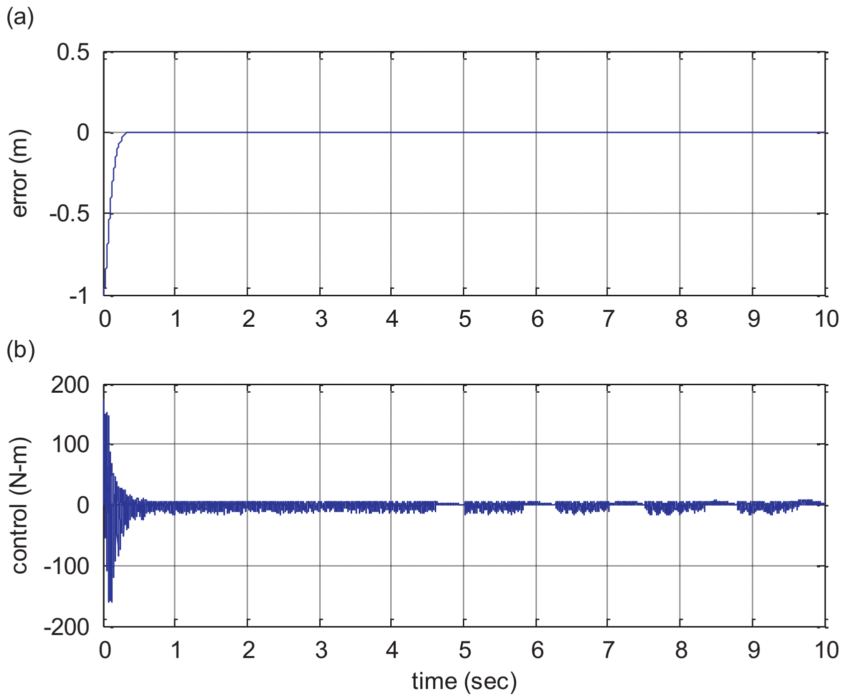

Fig. 5. Proposed control performance in case 2: (a) output error and (b) control effort.

approximation of the uncertain dynamics by using orthonormal basis functions under rigorous mathematical justification of closed loop stability. Simulation cases show effectiveness of the proposed design.

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